Cover page

Title: Multivariate Autoregressive Models for Local Damage Detection Using Small Clusters of Wireless Sensors

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ABSTRACT

Recent developments in the wireless sensing area have opened many possibilities for local damage detection in civil structures: when they become low-cost, it will be possible to put numerous sensors on a structure, and identify local damage. In this regard, it is important to embed algorithms on sensors to ensure network scalability: transmitting only the results of local computations reduces data exchange, so that communications are not a limiting factor.

It is believed that more accurate information about the structural dynamics can be obtained by using several channels in the analysis. Two methods are developed to obtain this information. The first method is an extension to multivariate models of an existing method using a two-stage AR-ARX model identification. The second method uses finite differences to rewrite the second-order equation of dynamics as an ARX model.

Numerical verification of the two approaches is performed on the benchmark problem designed by the ASCE task group on structural health monitoring. It is found that although the multivariate AR-ARX model method identifies global vibration modes of the structure, the local physical model method successfully detects damage for different models and excitation sources.

INTRODUCTION

In many countries in the world, a lot of infrastructures were built in the decades following the second world war. These infrastructures are now getting older, and assessing the structural integrity of infrastructure is becoming increasingly important.

The two major issues are to periodically check the health of the structures, and to make a rapid evaluation of a structure's health after major events such as earthquakes. It is important that the structural integrity be evaluated in real-time in such cases.

With recent developments in wireless sensors, it will soon be possible to install

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numerous low-cost sensors on a single structure. This would enable better local damage identification. However, in order to be able to increase the number of sensors, the network should be scalable. Therefore, the challenge is to develop damage identification methods that advantageously use the computational power of wireless sensors to keep communications to a minimum.

In 2001, Sohn and Farrar [7] presented a procedure to identify damage from the acceleration time histories of one sensor, using a two-stage AR-ARX model. In 2003, Lei et al. [2] modified this method to consider the influence of excitation variability on the damage-sensitive feature. They applied the method to a benchmark problem [1], and could detect and localize all medium and severe damage cases therein.

Lynch et al. [5, 4] embedded an algorithm implementing the original method of Sohn and Farrar on a wireless sensing platform, and showed that it would be far more energy-efficient to use such a procedure to compute a damage criterion and transfer only the damage identification result, than to transfer whole time histories.

In order to identify local damage, it is important that the models correspond to dynamic properties at the level of structural members. In this respect, it seems that exchanging time histories between adjacent sensors could improve the results, assuming that multivariate models more accurately describe the structural behavior in the members between sensors. For example, in a frame structure, the sensors of one structural element send their time histories to one of them which performs the damage analysis.

Two methods are investigated to obtain local structural information which is linked to damage. The first method is an extension to multivariate models of the two-stage AR-ARX method developed by Sohn and Farrar [7] and modified by Lei et al. [2]. The second method is developed specifically for frame structures. It uses finite differences to rewrite as an ARX(2,2) model the second-order dynamical equations of a node of the frame structure. Parameters of the ARX model are then identified for undamaged reference time histories and new time histories. A reference time history that is "closest" to the new time history is chosen, and the corresponding ARX model is applied to the new time history. The resulting residual error is chosen as damage-sensitive feature.

Both methods are applied for numerical verification to the ASCE Phase 1 Analytical Benchmark[1], with acceleration response data for various structural models and input sources, for several damage cases ranging from minor damage to severe damage.

METHODS

AR-ARX Method

The first damage identification method investigated in this research is an extension to multivariate models of the damage identification procedure presented by Sohn and Farrar [7] and modified by Lei et al. [2].

NORMALIZATION PROCEDURE

The first step is referred to as *normalization procedure* by Sohn and Farrar [7]. It is assumed that we have a pool of time histories which are acceleration responses (also called *outputs* of the system) of the undamaged structure, subject to various environmental conditions. These time histories are called the *reference database*, and denoted by $x(t)$. The reference signals $x(t) = \left[x_1(t)^T, \cdots, x_m(t)^T\right]^T$ are multidimensional quantities, where m is the number of output channels.

All signals are normalized so that they have zero mean and unit variance for each channel. Then, a multidimensional AR(p) model is built from each of the reference signals:

$$x(t) + A_1^x x(t-1) + \cdots + A_p^x x(t-p) = e_x(t) \tag{1}$$

The order p of the AR model is chosen by using Akaike's Information Criterion (AIC) [3].

A multidimensional AR(p) model with the same order p is built from $y(t)$, yielding coefficients $A_1^y, \cdots, A_p^y$ and a prediction error $e_y(t)$.

A reference signal $x_0(t)$ is then selected that is *closest* to $y(t)$, in the sense that the Euclidean distance between the AR(p) model coefficients is minimized:

$$D = \sum_{i=1}^{p} ||A_i^x - A_i^y||^2 \tag{2}$$

FEATURE EXTRACTION

Once the *closest* reference signal has been found from the database, a two-stage AR-ARX model is built from this reference signal.

The first stage consists in building a high-order AR model. This model is the same as the one built in the normalization procedure, Eq. (1). In this equation, $e_{x_0}(t)$ is the prediction error associated with this model for the signal $x_0(t)$. It is assumed that the estimated prediction error $e_{x_0}(t)$ of this AR model is a good estimate of the innovation process of the time series, i.e. of the input of the system.

It is then possible to build an ARX(a,b) model to describe the input-output behavior of the input $e_{x_0}(t)$ and output $x_0(t)$ signals:

$$\begin{aligned} x_0(t) \quad &+ \quad C_1^{x0} x_0(t-1) + \cdots + C_a^{x0} x_0(t-a) \\ &= \quad D_1^{x0} e_{x_0}(t-1) + \cdots + D_b^{x0} e_{x_0}(t-b) + \varepsilon_{x_0}(t) \end{aligned} \tag{3}$$

Here, the a and b are the orders of the ARX model. $\varepsilon_{x_0}(t)$ is the residual error of the ARX model, and will be used later. Sohn and Farrar [7], referring to Ljung [3], suggested to keep the sum of a and b smaller than p. In the following application, as the chosen AR model orders were between 30 and 40, orders of $a = 10$ and $b = 10$ were used. This is the second stage.

An AR(p) model has also been identified for the new signal $y(t)$, and provides us with an estimate $e_y(t)$ of the input signal corresponding to the output signal $y(t)$. The ARX model describing the input-output relationship between $e_{x_0}(t)$ and $x_0(t)$ is then applied to the input $e_y(t)$ and the output $y(t)$ to obtain a residual error $\varepsilon_y(t)$.

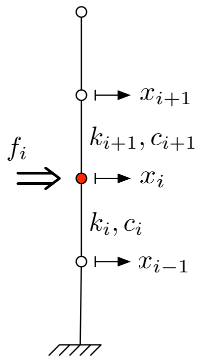


Figure 1. Shear model of a structure

If the new signal $y(t)$ does not belong to the same structural state as the reference signals, then even the ARX model for the closest signal $x_0(t)$ cannot accurately model the input-output relationship of the pair $(e_y(t), y(t))$ of input and output signals. Therefore, the residual error $\varepsilon_y(t)$ is chosen as damage-sensitive feature.

Following the modification introduced by Lei et al. [2], the ARX model of the selected reference signal $x_0(t)$ is applied to all the other reference signals $x(t)$. The residual error $\varepsilon_x(t)$ is calculated. The following quantity h' is finally computed along each dimension:

$$h'_i = \frac{\sigma(\varepsilon_{x,i}(t))}{\sigma(\varepsilon_{x_0,i}(t))} \tag{4}$$

The probability density function of h'_i is then evaluated using kernel density estimation methods described in Scott and Sain [6], and integrated to obtain the cummulative distribution function and compute a 97.5 % confidence threshold.

The variance ratio of the residual error for $y(t)$, $h = \sigma(\varepsilon_y(t))/\sigma(\varepsilon_{x_0}(t))$, is then tested against the threshold.

Local Physical Model Method

An approach is presented here that rewrites as an ARX(2,2) model the second-order equation of dynamics of a node of a frame structure. This model can be identified from acceleration measurements of the nodes that are connected by a structural member to the current node.

SHEAR MODEL EXAMPLE

Suppose that a structure is modelled as a uni-dimensional lump mass shear model, as in Figure 1. Each node i has a mass m_i. The structural member below node i is modelled by a stiffness coefficient k_i and damping coefficient c_i. The movement of each node is constrained in all directions except one direction, represented on Figure 1 by the displacement x_i for node i. Finally, a force f_i is applied to node i.

Under these conditions, the dynamics of node i are ruled by the following equation:

$$m_i\ddot{x}_i + c_i(\dot{x}_i - \dot{x}_{i-1}) + c_{i+1}(\dot{x}_i - \dot{x}_{i+1}) + k_i(x_i - x_{i-1}) + k_{i+1}(x_i - x_{i+1}) = f_i \tag{5}$$

After differenciating twice the equation, the first and second derivatives of the accelerations are approximated by finite differences

$$\frac{da_i}{dt}(t) = \frac{a_i(t) - a_i(t-T)}{T} \quad , \quad \frac{d^2a_i}{dt^2}(t) = \frac{a_i(t+T) - 2a_i(t) + a_i(t-T)}{T^2} \tag{6}$$

where T is the sampling interval. (5) becomes

$$\begin{aligned}
\frac{m_i}{T^2}a_i(t+T) &= \left(\frac{2m_i}{T^2} - \frac{c_i + c_{i+1}}{T} - (k_i + k_{i+1})\right)a_i(t) \\
&+ \left(-\frac{m_i}{T^2} + \frac{c_i + c_{i+1}}{T}\right)a_i(t-T) \\
&+ \left(\frac{c_i}{T} + k_i\right)a_{i-1}(t) - \frac{c_i}{T}a_{i-1}(t-T) \\
&+ \left(\frac{c_{i+1}}{T} + k_{i+1}\right)a_{i+1}(t) - \frac{c_{i+1}}{T}a_{i+1}(t-T) \\
&+ \frac{d^2f_i}{dt^2}(t)
\end{aligned} \tag{7}$$

Equation (7) can be thought of as a multivariate ARX(2,2) model

$$y(t) + A_1y(t-1) + A_2y(t-2) = B_1u(t-1) + B_2u(t-2) + e(t) \tag{8}$$

where $y(t) = a_i(t)$ is the system output, $u(t) = \begin{bmatrix} a_{i-1}(t) \\ a_{i+1}(t) \end{bmatrix}$ is the system input and $e(t)$ incorporates both the external force term $\frac{d^2f_i}{dt^2}$ and the model and measurement errors. Here, the notation T has been replaced by 1 for convenience of notations.

DAMAGE DETECTION

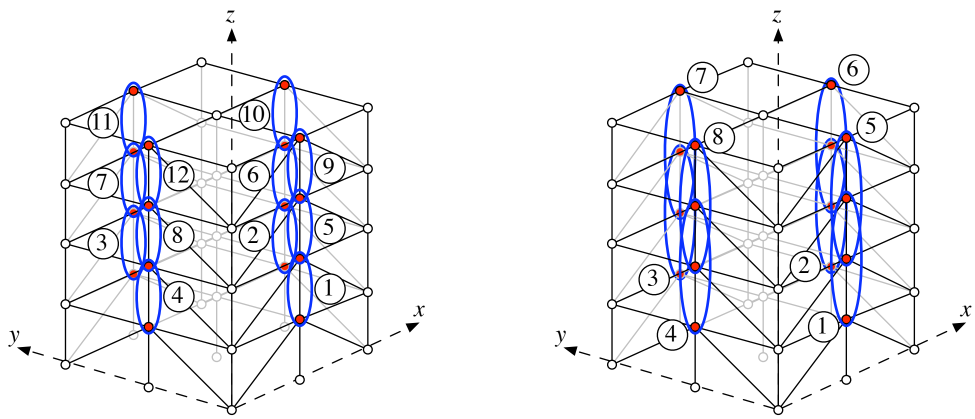
Based on this model, a damage detection methodology similar to the AR-ARX method is devised.

A pool of reference time histories $x(t)$ is built, each time history constituted of an input and an output signal: $x(t) = (x_u(t), x_y(t))$.

For the *normalization procedure*, the distance between the coefficients of the ARX(2,2) model in (8) is used to select the reference signal $x_0(t)$ that has *closest* environmental conditions.

Applying the ARX(2,2) model of the selected reference signal to a new signal $y(t)$ and to other reference signals $x(t)$, residual errors $e_y(t)$ and $e_x(t)$ are obtained.

Similarly to the AR-ARX method, a reference index $h' = \frac{\sigma(e_x(t))}{\sigma(e_{x_0}(t))}$ is computed. A threshold is built from its identified probability density function, and the new value $h = \frac{\sigma(e_y(t))}{\sigma(e_{x_0}(t))}$ is compared to the threshold.



(a) AR-ARX method
(b) local physical model method

Figure 2. ID number of the clusters accelerometer clusters

SIMULATION RESULTS

The ASCE Phase I Analytical Benchmark [1] is used as a test case. It has been studied with various methods, including the unidimensional AR-ARX method. The benchmark study therefore provides a useful common framework to test the two methods developed in this research, and to compare the results with those obtained by other methods.

It is a 4-story 2-bay by 2-bay steel frame structure with four concrete slabs on each floor. A *MATLAB* program provided by the ASCE Task Group on Health Monitoring was used to generate vibration responses of the structure[1] for 5 simulation cases combining different structural models - a 12-DOF finite element model or a 120-DOF finite element model, and different input sources - ambient vibration or a shaker on the roof. For each simulation case, the benchmark introduces 2 to 6 damage patterns.

In this study, the pool of reference signals comprises of 50 time histories of 10 seconds each, generated for the undamaged structure. 5 time histories are then generated and studied for each damage pattern. All acceleration time histories were acceleration data were downsampled from 1000Hz to 250 Hz using an 8^{th} order Chebyshev filter.

AR-ARX Method

Clusters of two accelerometers each are defined as in Figure 2(a) to allow for local damage identification. Each cluster is studied independently.

Case study 1 is a one-dimensional study in the y-direction, with the 12-DOF model, and ambient vibration as excitation. Only clusters along the y-direction are studied. Two damage patterns are introduced. Results for these damage cases are shown in Table I.A. The number of times that the index h crossed the 97.5 % confidence threshold are written for the two accelerometers of each cluster.

For damage pattern (i), all braces below the first floor are removed. Therefore, the index value should cross the threshold only for those clusters that have an accelerom-

[1] http://cive.seas.wustl.edu/wusceel/asce.shm/analyt_1.htm

TABLE I. DAMAGE IDENTIFICATION RESULTS FOR STUDY CASE 1; THE NUMBERS SPECIFY HOW MANY TIMES THE NULL HYPOTHESIS WAS REJECTED OUT OF THE 5 SAMPLES

(A) LOCAL STIFFNESS METHOD

damage patterns	(i)	(ii)	(vii)
cluster 2	0/5	5/5	0/5
cluster 4	0/5	5/5	0/5
cluster 6	0/5	5/5	0/5
cluster 8	0/5	5/5	0/5

(B) AR-ARX METHOD

damage patterns	(i)	(ii)	(vii)
cluster 2	5/5 5/5	5/5 5/5	0/5 0/5
cluster 4	5/5 4/5	5/5 5/5	0/5 0/5
cluster 6	5/5 4/5	5/5 5/5	0/5 0/5
cluster 8	5/5 1/5	5/5 5/5	0/5 0/5
cluster 10	3/5 0/5	3/5 3/5	0/5 0/5
cluster 12	2/5 1/5	5/5 3/5	0/5 0/5

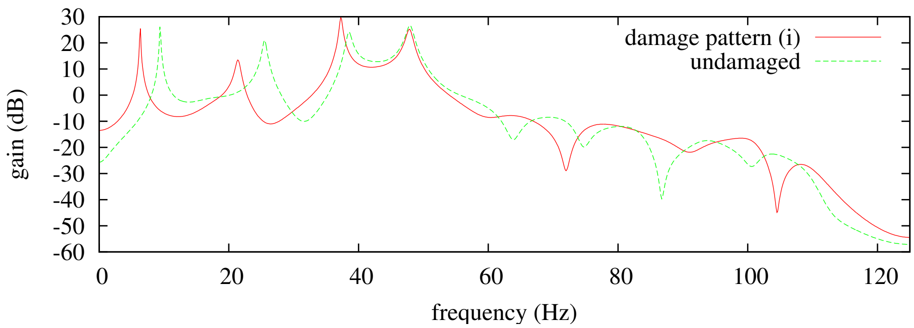


Figure 3. Gain of the transfer function of the identified AR model between the two accelerometers (study case 1, cluster 8)

eter on the first floor, namely clusters 2 and 4. However, results indicate that damage is found also in other clusters.

For damage pattern (ii), all braces between the second and third floors are removed. In this case, all clusters touch members that have been removed, and should report damage. However, there are some time histories for which damage is not found.

Figure 3 shows the gain of the transfer function between the two accelerometers of cluster 8. The first four structural modes are identified. The decrease in natural frequency of the global modes is thus picked up by the model, and is thought to be the source of the previous results.

Although it would be possible to filter out unwanted frequency components for each cluster so that the identified model describes only local properties, this method was not investigated, because it would no longer be a low-cost damage identification method.

Local Physical Model Method

For the local physical model method, clusters comprising of three accelerometers each where defined as in Figure 2(b), following the above-mentionned shear model. In each cluster, data from the middle accelerometer are defined as output, and data from the top and bottom accelerometers are defined as input.

Table I.B shows the results for study case 1. With damage pattern (i), all braces below the first floor are removed. These braces are contained in none of the defined clusters. Accordingly, the indicator does not exceed the threshold. For damage pattern (ii), in addition to damage pattern (i) all braces between the second and third floors are removed. Accordingly, all clusters containing these braces, namely all clusters studied here, identify damage.

Similar results were obtained for the other one-dimensional analyses in the y-direction, which have different models and inputs. A different local physical model is being investigated for 3-dimensional studies.

CONCLUDING REMARKS

Two decentralized damage identification methods involving autoregressive models are developed and investigated. The target application is low-cost real-time local damage identification with wireless sensors. Both methods were applied to acceleration data from the ASCE Phase I Analytical Benchmark.

With the AR-ARX method, it is found that although damage is detected, it is detected not only in the clusters that are near the damage, but also in other clusters. Although it would be possible to filter out unwanted frequencies and identify only local dynamic properties, this was not investigated further.

With the local physical model method, damage is successfully detected and localized for different models and excitation sources. However, the 3-dimensional vibration cases of the Benchmark need to be analyzed with the generalization of the method to provide for a 3-dimensional local physical model. This is currently under investigation.

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